This exercise is adapted from one developed by D. Scott Birney at Wellesley College

Proper Motion of a Star

In this exercise you will measure the proper motion of Barnard’s Star and use it, with a given parallax and radial velocity, to determine the space motion of the star.

Background information

In 1718 Edmund Halley had noted that some stars were not fixed, but appeared to move in the sky relative to other stars. Arcturus in Boötes and Sirius in Canis Major were the first stars detected by Halley to have proper motion.

Proper motion is the angular change in position of a star across our line of sight, measured in arc seconds per year, and symbolized with the Greek letter “μ” μ. Proper motion is generally measured by taking photographs several years apart and measuring the movement of the image of a star with respect to more distant background stars over that time period. Usually decades must elapse between successive photographs before a reliable measurement can be made.

The star with the largest proper motion was discovered by E. E. Barnard in 1916 at Yerkes Observatory. This star, now called Barnard’s Star, is a 9.5 magnitude star located in the constellation Ophiuchus. Its proper motion is so much larger than that of any other star that it is considered to be virtually a “runaway” star. Even so, the angular velocity is small enough that measurements must be made from photographs taken many years apart. The negatives of Barnard’s Star provided in this lab were taken in 1924 and 1951 respectively.

Since proper motion is an angular velocity, we also need to know the star’s distance to find its real velocity across our line of sight, called its tangential velocity ($v_t$). Two stars with the same proper motions can have vastly different tangential velocities, as shown in the example in figure below.
Since stellar parallaxes are usually tabulated rather than stellar distances, tangential velocity can be calculated from the following equation

\[ v_t = \frac{4.75 \times \mu}{p} \]  

Equation 1

where \( v_t \) is the tangential velocity in km/sec
\( \mu \) is the proper motion in seconds of arc/planet
\( p \) is the parallax in seconds of arc

Radial velocity \( (v_r) \) describes the motion of a star along our line of sight. A negative radial velocity indicates motion toward us, and positive motion away.

We still need one more piece of information to know the velocity and direction a star moves in space, called its space motion or space velocity \( (v) \). The radial velocity \( (v_r) \) is added vectorially to the tangential velocity \( (v_t) \) (at right angles) to give us the total space velocity \( (v) \).

The magnitude of special velocity \( (v) \) can be calculated using the Pythagorean Theorem.

\[ v = \sqrt{v_r^2 + v_t^2} \]  

Equation 2

Space motion allows us to study the dynamics and interactions of groups of stars, and to see how the arrangement of our local group of stars is changing over the centuries.

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PROPER MOTION of BARNARD'S STAR

SEPT. 4, 1951

.... 4 .... 1
.... B .... 2
.... .... .... 3

SEPT. 18, 1924

.... 4 .... 1
.... B .... 2
.... .... .... 3

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Procedure

Part 1. PROPER MOTION

Place a transparency of one of the two images taken of Barnard’s Star and the surrounding field on top of the other and align stars 1, 2, 3, and 4. Note that the stars were exposed more than once; therefore use only the right-hand image of each star. While holding the transparency in place, measure the distance between the two positions of Barnard’s Star to a tenth of a millimeter.

\[ D = \text{_______________ mm} \]

Convert this distance to the seconds of arc using the following conversion factor: 
24.52 seconds of arc = 1 mm. Show your calculations.

\[ D = \text{_______________ seconds of arc} \]

These two photographs were taken exactly \textbf{26.96 years apart}. Calculate the proper motion of Barnard’s Star in seconds of arc per year. Show your calculations.

\[ \mu = \text{_______________ seconds of arc/year} \]

\[ \mu = \frac{D(\text{in seconds of arc})}{26.96 \text{ years}} = \]

Part 2. SPACE MOTION

The parallax of Barnard’s Star has been measured to be \( p = 0.545 \text{ seconds of arc} \). Using Equation 1, find the tangential velocity of Barnard’s Star in km/sec. Show your calculations.

\[ V_t = \text{_______________ km/sec.} \]

\[ V_t = \frac{4.75 \times \mu}{p} = \]

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Barnard’s Star has a radial velocity of $V_r = -108 \text{ km/sec}$. Calculate the magnitude of the star’s space velocity by using Equation 2. Show your calculations.

$$V = \text{__________________________ km/sec}$$

**Part 3. CLOSEST APPROACH to the SUN**

Since Barnard’s Star’s negative radial velocity indicates that it is moving generally toward us, we can find the epoch of its closest approach to the Sun and discover just how close that approach will be.

On a piece of graph paper from a point representing the Sun, draw a line to represent the direction of Barnard’s Star. Calculate the distance to Barnard’s Star in parsecs from the parallax-distance relation (distance = 1/parallax).

$$d = \frac{1}{p} = \text{_______________ pc.}$$

Using a convenient scale (such as 10 cm = 1 pc), mark a point at the proper distance from the Sun to represent Barnard’s Star.

Along the line and from Barnard’s Star, draw an arrow representing the radial velocity of the star (a scale of $\frac{1}{2}$ mm = 1 km/sec works well).

Again from Barnard’s Star draw another arrow (at right angles) representing the tangential velocity, using the same scale as for $V_r$.

Complete the rectangle to find the direction and magnitude of the space velocity of Barnard’s Star. Measure the magnitude of the space velocity of Barnard’s Star and converts it to km/sec using the same scale as you used for the radial and tangential velocities ($\frac{1}{2}$ mm = 1 km/sec).

$$V = \text{__________________________ km/sec}$$

How does this value of the space velocity compare to the one calculated in part 2 by using formula 2.

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To find the point of Barnard’s Star’s closest approach to the Sun extend the diagonal of the space velocity to show the path the star will take. Using a ruler and a protractor, find the point along this line where Barnard’s Star is closest to the Sun. A line connecting the sun and point of closes approach should be perpendicular to the path of Barnard’s Star.

Measure the closest approach distance in cm and convert to parsecs using the same scale factor 10 cm =1pc. How close will Barnard’s Star come to the Sun?

\[ D_{\text{closest to the Sun}} = \ \text{_______________ pc} \]

By measuring from your diagram, determine how far (in pc) Barnard’s Star has to travel from its present position to its point closes to the Sun. (don’t forget to convert centimeters to parsecs using a scale of 10 cm =1pc)

\[ D_{\text{from present position to point closest to the sun}} = \ \text{_______________ pc} \]

From the velocity and distance it has to travel we can calculate how long it will take to get nearest to us: time = distance/velocity.

\[ t = \ \text{_______________ pc/(km/sec)} \]

Using the conversion factor \( 1 \text{ km/sec} = 1.02 \times 10^{-4} \text{ pc/century} \), how far in the future will it be when Barnard’s Star is closest to the Sun? Show your calculations.

\[ t = \ \text{_______________ centuries} \]

Given today’s date, calculate the year that Barnard’s Star will be closest to the Sun

\[ \text{_______________ AD.} \]

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Questions

1. At present the closest star to the Sun is Promixa Alpha Centauri at 1.33 pc. How will Barnard’s Star compare to this at its closest approach?

2. How do we know that the four reference stars don’t have proper motions of their own?