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1. In simplified form $\dfrac{\left(a^2\right)^3\left(b^{-2}\right)^3}{(ab)^2 c^{-3}}$ is

   a. $\dfrac{c^3}{a^2 b^2}$

   b. $\dfrac{a^4}{b^8 c^3}$

   c. $\dfrac{a^3}{b^7 c^3}$

   d. $\dfrac{a^4 c^3}{b^8}$

   e. $\dfrac{a^3 c^3}{b^7}$

2. Consider the number $\dfrac{a}{b}$ where a is an integer, b is a non-zero integer, a and b have no common natural number factors other than 1, and 7 is a prime factor of b. Which of the following statements is true of the decimal representation of $\dfrac{a}{b}$?

   a. It is a terminating decimal.
   b. It is a repeating decimal.
   c. It is a decimal that neither terminates nor repeats.
   d. There is not enough information given to determine the nature of the decimal representation of the number $\dfrac{a}{b}$.
   e. There is not a decimal representation of the number $\dfrac{a}{b}$.

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3. Consider the number $\frac{C}{d}$ where C is the circumference of a circle in inches and d is the diameter of the same circle in inches. Which of the following statements is true of the decimal representation of $\frac{C}{d}$?

   a. It is a terminating decimal.
   b. It is a repeating decimal.
   c. It is a decimal that neither terminates nor repeats.
   d. There is not enough information given to determine the nature of the decimal representation of the number $\frac{C}{d}$.
   e. There is not a decimal representation of the number $\frac{C}{d}$.

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4. Sam, Armani, Jo, Thandiwe, and Gabriela were the five participants in a contest to program a robot to travel along a straight line, with the winner being the person whose robot travelled the greatest distance in 9 seconds. Each robot traveled for at least a full nine seconds. Below is information about how each contestant's robot performed. Who won the contest?
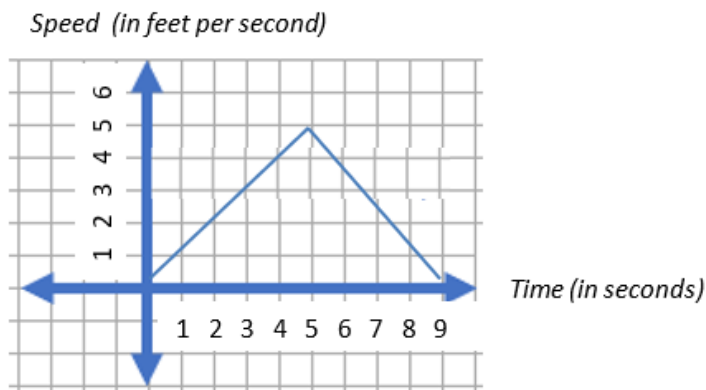
Sam's robot:

Speed (in feet per second)

Time (in seconds)

1 2 3 4 5 6 7 8 9
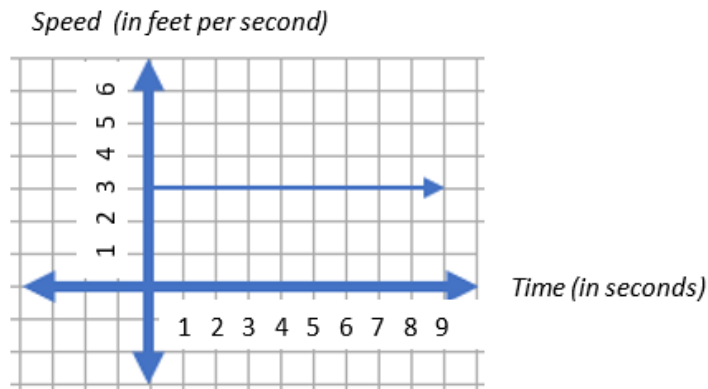
Armani's robot: | $d = 5t$ where t is the time in seconds and 5 represents 5 feet per second.

Jo's robot:

Time (in seconds)	Distance (in feet)
0	0
1	4
2	8
3	12
4	16
Etc.	

Speed (in feet per second)

Thandiwe's robot:

Time (in seconds)

1 2 3 4 5 6 7 8 9

Gabriela's robot:

Time (in seconds)	Speed (in feet per second)
0	0
1	.5
2	1
3	1.5
4	2
Etc.	

a. Sam   b. Armani   c. Jo   d. Thandiwe   e. Gabriela

5. Which of the following four sets of measurements could be the side lengths of a right triangle?   (The numbers are not listed in any particular order.)

$$\text{Set 1:} \quad 1 \ in, \quad 2 \ in, \quad 3 \ in$$

$$\text{Set 2:} \quad 1\frac{1}{2} in, \quad \sqrt{8\frac{1}{2}} \ in, \quad 2\frac{1}{2} \ in$$

$$\text{Set 3:} \quad 2 \ in, \quad 1 \ in, \quad \sqrt{3} \ in$$

$$\text{Set 4:} \quad 26 \ in, \quad 24 \ in, \quad 10 \ in$$

     a. All four sets
     b. Only set 2, set 3, and set 4
     c. Only set 2 and set 3
   d. Only set 1 and set 4
     e. Only set 4

6. In the diagram to the right, quadrilateral ABCD is a square with side length 7. M_1 is the midpoint of side AB and M_2 is the midpoint of side AD. What is the area of quadrilateral M_1BDM_2?

     a. $\dfrac{49}{4}$ square units

     b. $\dfrac{49}{3}$ square units

     c. $\dfrac{(3)(49)}{8}$ square units

     d. $\dfrac{(5)(49)}{12}$ square units

     e. $\dfrac{(3)(49)}{5}$ square units
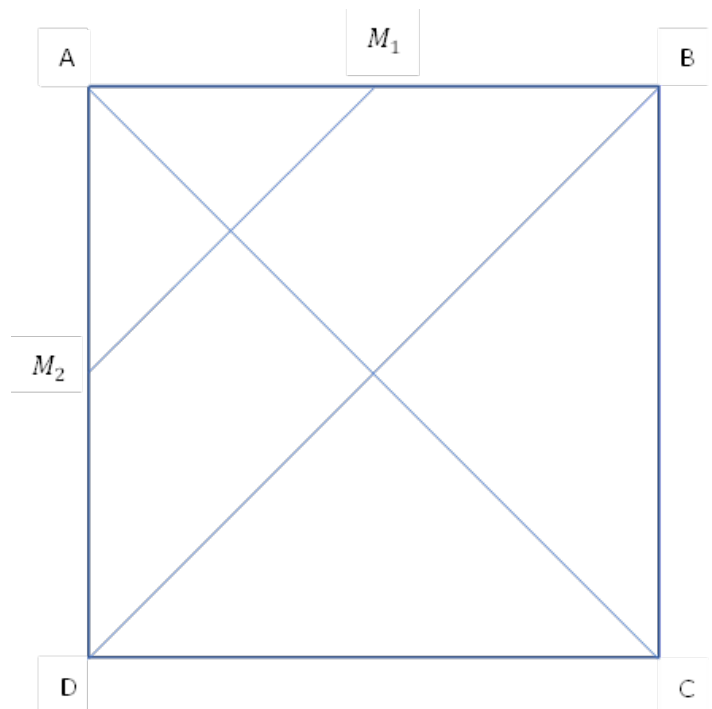
7. Eli runs 100 yards down the sideline of a football field in 20 seconds. The total number of yards he has run at time t where $0 \leq t \leq 20$ is given by the graph below. What is Eli's average speed in the time interval between 5 seconds and 10 seconds?

Distance (in yards)
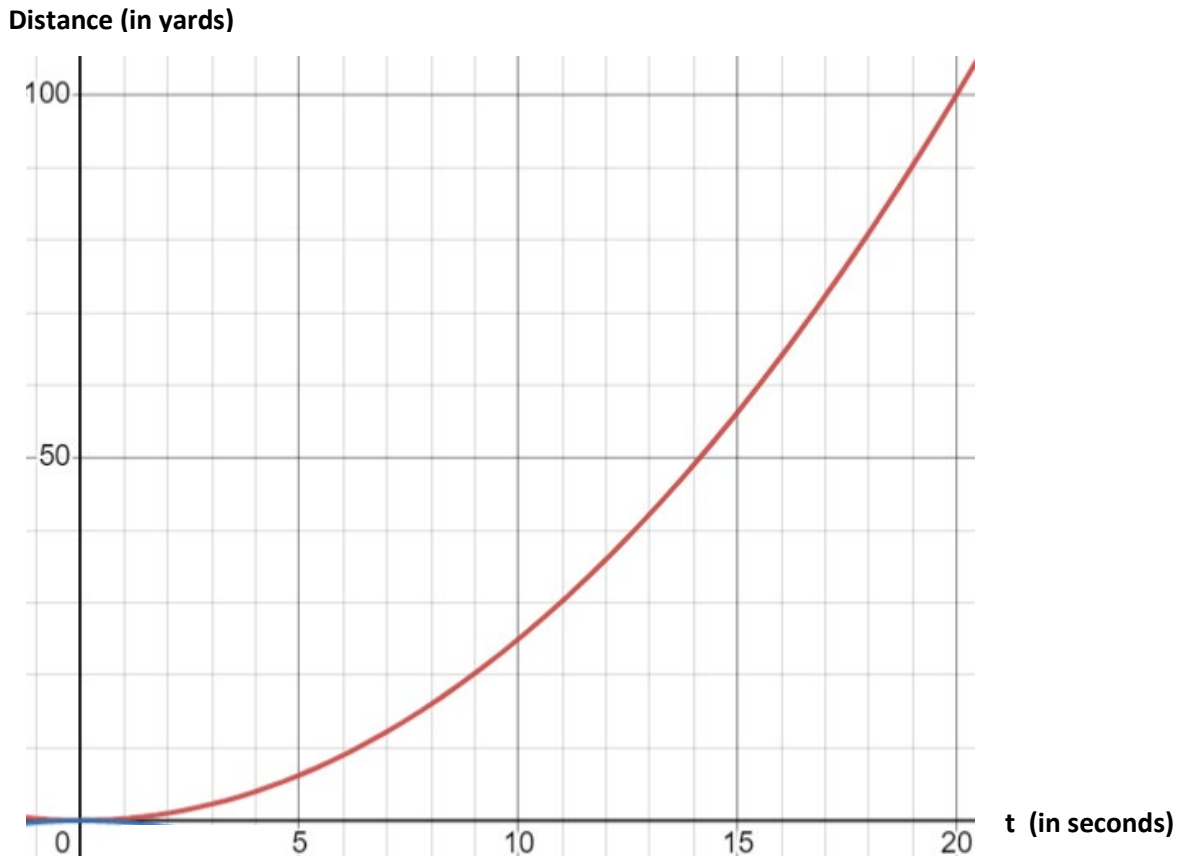
    a. 2.5 yards per second
    b. 3 years per second
    c. 4 yards per second
    d. 5 yards per second
    e. 6 yards per second

8. The equation $\left(\frac{5}{4}\right)(x-1) + \left(\frac{3}{5}\right)(x-1) = x - 1$ has
    a. No solutions.
    b. Exactly one solution.
    c. Exactly two solutions.

    d. Infinitely many solutions.

    e. It is impossible to determine.

9. Teresa plays the game Wordle daily. She has played 287 times. Thirteen (13) times she has not gotten the word at all, but the data below shows that in the remaining 274 times, she has gotten the word in six tries 28 times, in five tries 60 times, in four tries 109 times, in three tries 62 times, in two tries 15 times, and she has never gotten the correct word in one try. Considering only the data in the graph below (the 274 tries in which she has gotten the word in six or fewer tries), compare the **mean** number of tries it takes Teresa to get the correct word with the **median** number of tries it takes her to get the correct word with the **mode**. Which of the following statements is true?

GUESS DISTRIBUTION

1	0
2	15
3	62
4	109
5	60
6	28
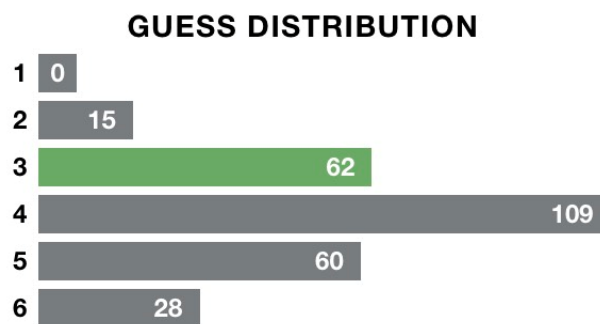
    a. mean > median > mode

    b. mean > median = mode

    c. mode > median > mean

    d. mode = median > mean

    e. median > mean > mode

10. At Top Notch Shipping it costs $\$a$ to ship a package plus *b cents per ounce*. At Nifty Shipping it costs $\$c$ to ship a package plus *d cents per ounce*. Thus which company is more economical depends on the weight of the package being shipped. When a package is a certain weight, the companies are equally economical. What is this weight?

    a. $100\left(\dfrac{a-c}{d-b}\right)$ ounces

    b. $\dfrac{a-c}{d-b}$ ounces

    c. $100\left(\dfrac{d-b}{a-c}\right)$ ounces

    d. $\dfrac{a}{b} - \dfrac{c}{d}$

    e. $\dfrac{a}{d} + \dfrac{c}{b}$ ounces

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11. In Large High School, there are 50 homerooms. Each homeroom has a President and a Secretary. Each of the 100 total Presidents and Secretaries is assigned a different one of the natural numbers 1 – 100. One hundred (100) balls, each having a different one of the natural numbers 1 – 100 on them, are placed in a Bingo cage. The handle on the Bingo cage is turned once, spilling out a ball at random. The person who had been assigned the number of that ball wins a laptop. Then the handle is turned again, spilling out another ball. (The first ball that came out was not put back in before the handle was turned the second time.) The person who had been assigned the number of this second ball wins a $100 gift certificate. What is the probability that the two winners are from the same homeroom?

a. $\dfrac{1}{9900}$ b. $\dfrac{1}{4950}$ c. $\dfrac{1}{198}$ d. $\dfrac{1}{99}$ e. $\dfrac{1}{50}$

12. If $(4,5)$ is a point on the graph of a line, and the slope of the line is $\dfrac{1}{6}$, which of the following is another point on the line?

a. $\left(\dfrac{2}{3},\dfrac{5}{6}\right)$ b. $\left(\dfrac{25}{6},\dfrac{31}{6}\right)$ c. $(10, 6)$ d. $(5,11)$ e. $(3,-1)$

13. Solve the equation $a(bx + ad) = c(y - a)$ for y.

    a. $y = \left(\dfrac{a}{c}\right)(bx + ad + c)$

    b. $y = \left(\dfrac{a}{c}\right)(bx + ad + a)$

    c. $y = \left(\dfrac{c}{a}\right)(bx + ad + a)$

    d. $y = \dfrac{abx}{c} + \dfrac{2ad}{c} + a$

   e. $y = \dfrac{abx}{c} + \dfrac{ad}{c} + \dfrac{a}{c}$

14. When graphed, the ordered pairs (x, y) in the table below lie on a line. What is the x-coordinate of the point on this line whose y-coordinate is -3?

x	y
-1	4
3	9
5	11.5

a. $\dfrac{-33}{5}$　　　　b. $\dfrac{3}{2}$　　　　c. $\dfrac{-39}{4}$　　　　d. $\dfrac{12}{5}$　　　　e. $\dfrac{-31}{5}$

15. Suppose a quantity Q_0 decreases by 40%, and then the resulting quantity increases by 40%. How much results?

a. $0.56\, Q_0$　　　　b. Q_0　　　　c. $0.96\, Q_0$　　　　d. $0.94 Q_0$　　　　e. $0.84 Q_0$

16. The radius of the base of a cone is the same length as the radius of the base of a right circular cylinder. The volume of the cone is also equal to the volume of the cylinder. Which of the following statements is true?

   a. The height of the cone is $\sqrt{3}$ times that of the cylinder.
   b. The height of the cylinder is $\sqrt{3}$ times that of the cone.
   c. The height of the cone is $\frac{4}{3}$ times that of the cylinder.
   d. The height of the cone is 3 times that of the cylinder.
   e. The height of the cylinder is 3 times that of the cone.

17. If the radius of a sphere triples, what is true about the volume?
   a. The new volume is 3 times the original volume.
   b. The new volume is 4 times the original volume.
   c. The new volume is 6 times the original volume.
   d. The new volume is 9 times the original volume.
   e. The new volume is 27 times the original volume.

18. P and Q are directly proportional. When P increases by 3, then Q increases by 2; and when Q increases by 2, then P increases by 3. Which of the three graphs below, I, II, III, could be a graph showing the relationship between these two quantities?
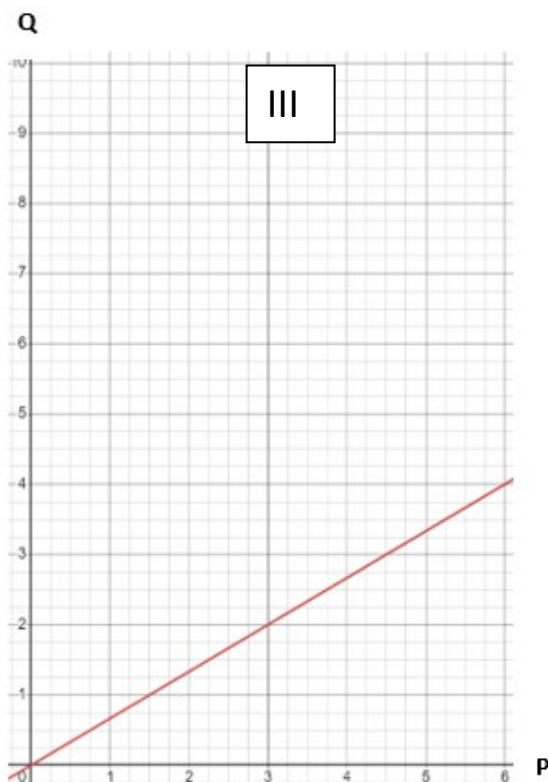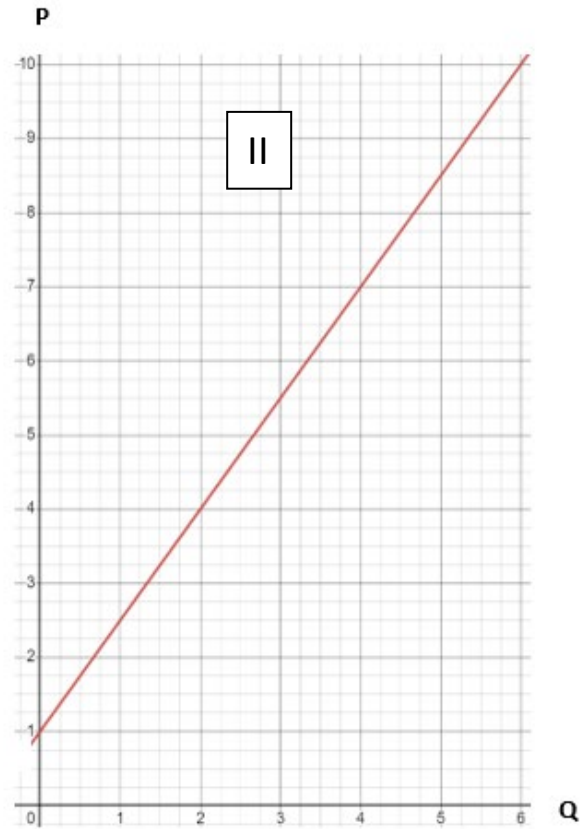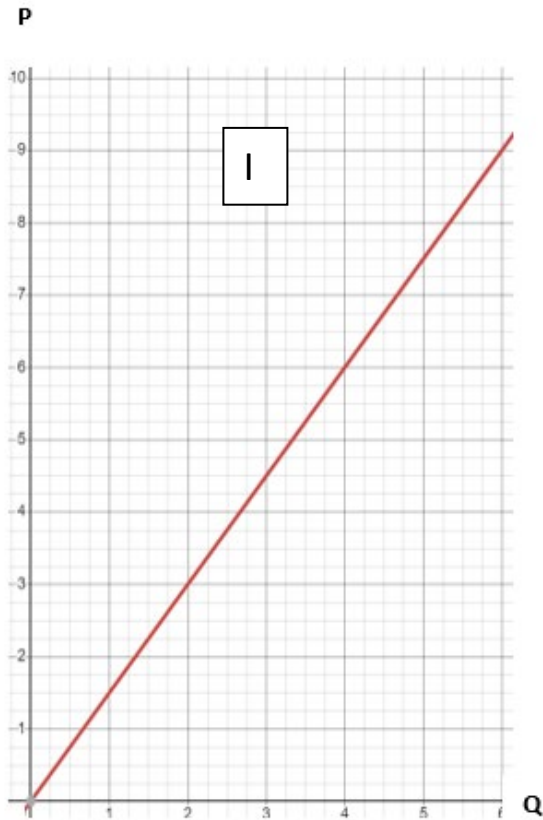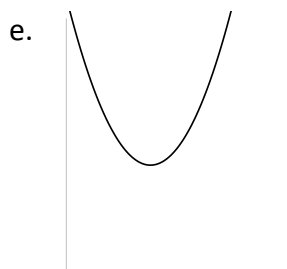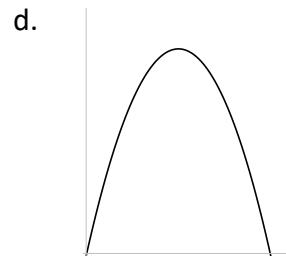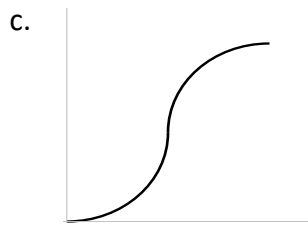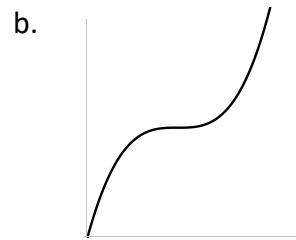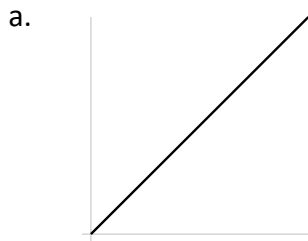

a. I and II and III
b. I and III only
c. II and III only
d. II only
e. I only

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19. Water is being poured into the container pictured below at a constant rate.
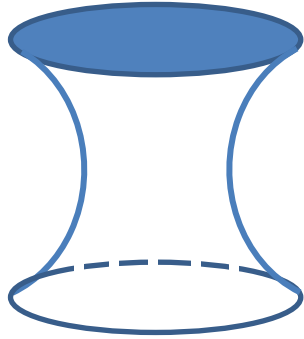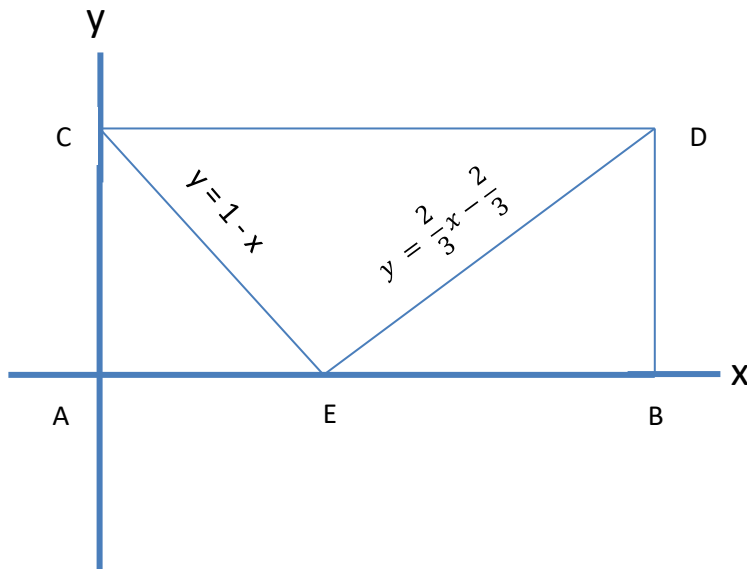

Which of the following could be the graph of the height of water in the container as a function of time?   (The x axis represents time, and the y-axis represents height of the water.)

a.

b.

c.

d.

e.

20. Quadrilateral ABDC is a rectangle. Find the area of triangle CED.

a. $\dfrac{5}{4}$ square units

b. $\dfrac{7}{4}$ square units

c. $\dfrac{3}{2}$ square units

d. $\sqrt{2} + \dfrac{\sqrt{13}}{2}$ square units

e. $\dfrac{\sqrt{26}}{4}$ square units

21. Consider the following sequence of 5 whole numbers, where each number is the sum of the previous two numbers: 3, 12, 15, 27, 42
    If you want 100 to be in the sequence, which of the following first two seed numbers would make this possible? (The seed numbers are the first two whole numbers in the sequence).
    a. 2, 5
    b. 3, 9
    c. 3, 11
    d. 4, 10
    e. 4, 12

22. The table lists percentages of women in state legislatures for the time period 1975 – 1995.

| Year	Percent
1975	8.0
1977	9.1
1979	10.3
1081	12.1
1983	13.3
1985	14.8
1987	15.7
1989	17.0
1991	18.3
1993	20.5
1995	20.7

The line of best fit for this data is $y = 0.66045\,x - 1296.5$. What is the meaning of the slope?

a. For the time period, 1975 – 1995, the percentage of women in state legislatures increased by about 0.66% per year.
b. For the time period, 1975 – 1995, the percentage of women in state legislatures increased by about 0.66% every two years.
c. Over the time period 1975 – 1995, the percentages of women in state legislatures averaged about 66 percent.
d. For the time period, 1975 – 1995, the percentage of women in state legislatures increased by about 66% per year.
e. For the time period, 1975 – 1995, on average the percentage of women in state legislatures increased by about 66%/20 or 3.3%.

23. A Pringles can has a height of 10 inches. A mini-Coke can has a height of 3.5 inches. The diameter of the Pringles can is twice that of the mini-Coke can. If four mini-Coke cans are placed into the Pringles can what percentage of the Pringles can is empty?

    a.   8.75%
    b.   30%
    c.   35%
    d.   65%
    e.   91.25%

24. If quadrilateral ABCD is rotated 90 degrees clockwise about point C, what are the coordinates of the image of point B?


    a. (1, -2)
    b. (2, -1)
    c. (5, -2)
    d. (4, 0)
    e. (4, -1)
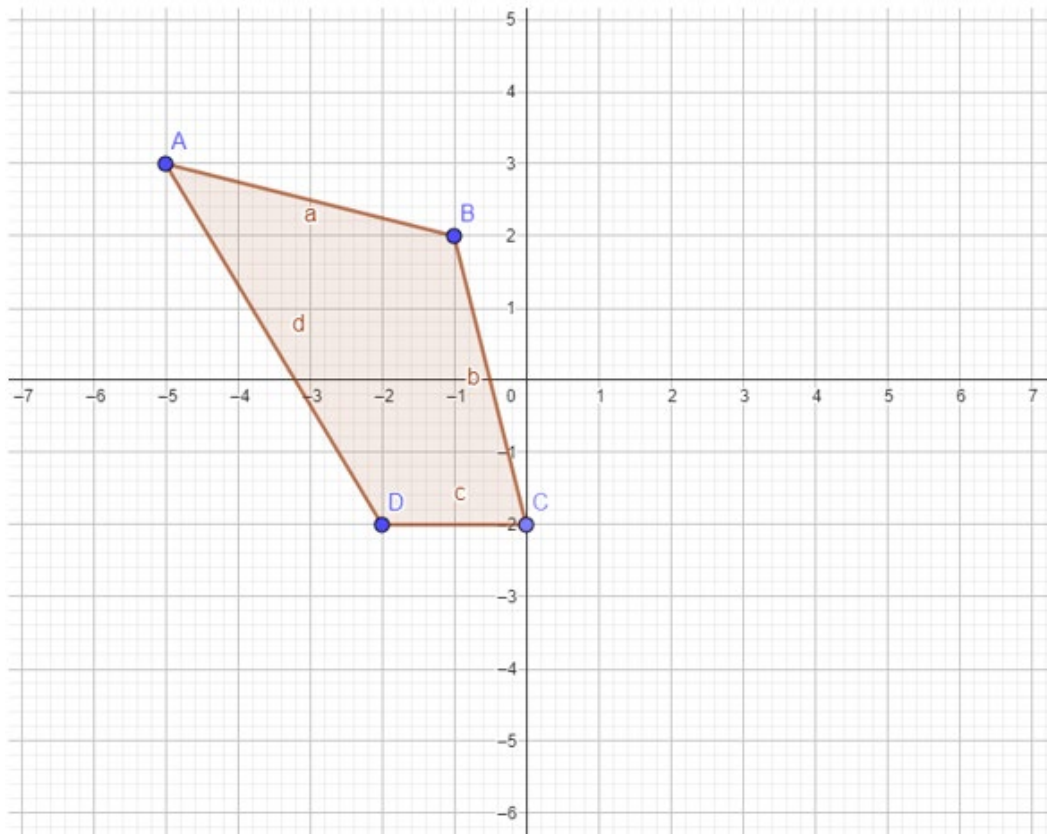
25. In the figure below, tangram pieces have been put together to form a picture of a cup. If the side of the square is 1 centimeter, what is the perimeter of the cup? (Tangram Piece Relationships: 2 small triangles make a square; 2 small triangles make a parallelogram; 2 small triangles make a medium triangle; 2 medium triangles make a large triangle). All triangles are right isosceles triangles.

  $a.\ 12 + 2\sqrt{2}$

  $b.\ 11 + 2\sqrt{2}$

  $c.\ 10 + 2\sqrt{2}$

  $d.\ 12 + \sqrt{2}$

  $e.\ 10 + \sqrt{2}$
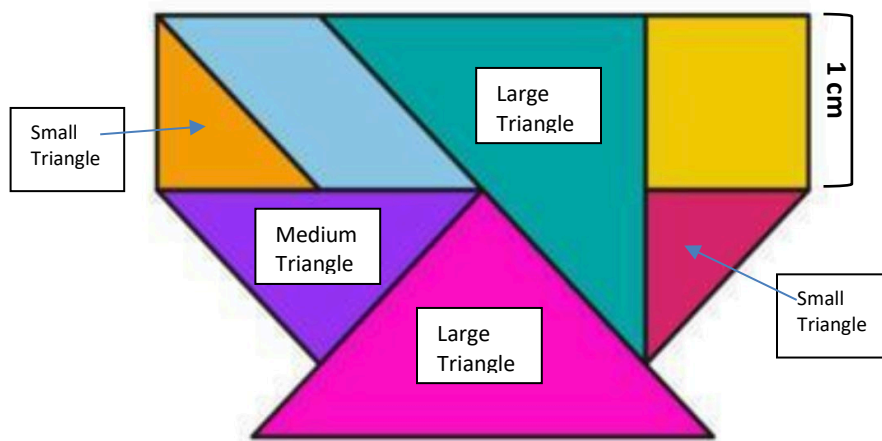
26. There are 14 different types of doughnuts made each day at the doughnut shop. The workers are told to put doughnuts of three different types into a three-pack. How many different kinds of three-packs can be made each day?

    a. 42
    b. 364
    c. 1,092
    d. 1,372
    e. 2,184

27. You have three cards in front of you. One card is labeled 44, one is labeled 59 and one is labeled 38. On the back of each of the cards is a prime number. The prime numbers on the back of the three cards are all different. The sum of the number on the front and the number on the back is the same for each card. Which of the statements are true about the prime numbers on the back of the cards?

    a. The prime numbers must be sequential.
    b. All of the prime numbers must be odd.
    c. The product of the prime numbers on the back of the cards is odd.
    d. All of the above are true.
    e. None of the above are true.

28. Consider the system of equations $\begin{cases} ax + by = c \\ 2x + 3y = 5 \end{cases}$ where a, b, c are real numbers.

    If the system of equations has no solutions, then what must be true?

    a. $\dfrac{a}{2} = \dfrac{b}{3} = \dfrac{c}{5}$

    b. $\dfrac{a}{2} = \dfrac{b}{3} \neq \dfrac{c}{5}$

    c. $\dfrac{a}{2} \neq \dfrac{b}{3} \neq \dfrac{c}{5}$

    d. $10a = 15b \neq 6c$

    e. $15a = 10\,b = 5c$

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29. Suppose Lynn wants to mix exactly x liters of a 12% salt solution with exactly y liters of a 20% salt solution to end up with exactly 6 liters of a 15% salt solution. What is the value of x?

    a. $3\frac{3}{4}$ liters

    b. $2\frac{1}{4}$ liters

    c. $3\frac{1}{4}$ liters

    d. $2\frac{3}{4}$ liters

    e. $4\frac{1}{4}$ liters

30. A small bakery bakes only one size of cookies. Before opening their store on February 12, 2023, they baked some cookies and put all of these in their large display case. They always line their cookies up in straight lines in the display case. On February 12, 2023, after a few attempts they realize that the only way the lines have the same number of cookies in them is if the cookies are placed in lines of 11 cookies. The table below shows the bakery's attempts to line the cookies up.

Number of cookies attempted to be put in each line	Cookies left over
2	1
4	3
5	4

What is the smallest number of cookies the company baked on February 12, 2023, such that all the information above is true?

    a.   44 cookies
    b.   99 cookies
    c.   121 cookies
    d.   209 cookies
    e.   319 cookies