TENNESSEE MATHEMATICS TEACHERS ASSOCIATION

SIXTY-SIXTH ANNUAL MATHEMATICS CONTEST

2024

Algebra II/Integrated Math III

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Scoring Formula: $4 \times$ (Number Right) $-$ (Number Wrong) $+ 40$

Directions:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

1. What is the value of $3^3 - 2^2[-8(-7+4)]$?
   a. 69
   b. −325
   c. -69
   d. 325
   e. -256

2. Eli wants to take an advanced mathematics course in his senior year. To do so, he needs at least a 93 average (mean) on a series of five tests in his junior year. His scores on the first four tests were 89, 96, 87 and 98. What is the lowest score that he needs on Test 5 to obtain at least a 93 average (mean) on the five tests?
   a. 95
   b. 90
   c. 88
   d. 97
   e. 99

3. What value for t, rounded to the nearest hundredth, satisfies the equation $3e^{-2t} = 175$?
   a. 2.03
   b. -29.17
   c. 29.17
   d. -2.03
   e. 7.68

4. What are all the zeros of $f(x) = x^3 - 2x^2 - 4x + 8$?
   a. $x = 0, 2$
   b. $x = 2, -2$
   c. $x = 2$
   d. $x = -2$
   e. $x = 3, -4$

5. Given $f(x) = 2x^2 - 3$ and $g(x) = 3x + 7$, what is $(f - g)(2)$?
   a. 8
   b. 18
   c. -18
   d. -8
   e. 65

6. A high school is planning on putting on a musical performance. There are 18 singers auditioning for the musical, and the director is looking for two singers for a duet. What is the probability that Alicia and Juan are the two singers selected?
   a. 0.0033
   b. 306
   c. 153
   d. 0.0556
   e. 0.0065

7. What is the leading term of $p(x) = -x^3(2x + 1)^2(3x - 5)$?
   a. $-12x^6$
   b. $12x^6$
   c. $-12x^7$
   d. $-x^6$
   e. x^6

8. If θ is a second-quadrant angle with $\sin \theta = \frac{6}{11}$, what is the value of $\tan \theta$?
   a. $\dfrac{6}{\sqrt{85}}$
   b. $-\dfrac{6}{\sqrt{85}}$
   c. $\dfrac{\sqrt{85}}{6}$
   d. $-\dfrac{\sqrt{85}}{6}$
   e. 1

9. What value for x satisfies the equation $-3[4x - 3 + 2(2x + 1)] = 7 - 7x$?
   a. $-\dfrac{4}{17}$
   b. $\dfrac{4}{17}$
   c. $-\dfrac{17}{4}$
   d. $\dfrac{17}{4}$
   e. $-\dfrac{12}{17}$

10. To the nearest tenth, what is the distance between the x-intercept and the y-intercept of $y = \dfrac{15-5x}{2x-3}$?

   a. 6.8

   b. 7.3

   c. 1.5

   d. 5.1

   e. 5.8

11. What values for x satisfy the inequality $\left|\dfrac{2x-3}{4}\right| > 1$?

   a. $\left\{x : x < -\dfrac{1}{2}\right\}$

   b. $\left\{x : x > \dfrac{7}{2}\right\}$

   c. $\left\{x : x < -\dfrac{1}{2} \text{ or } x > \dfrac{7}{2}\right\}$

   d. $\left\{x : x < \dfrac{1}{2} \text{ or } x > \dfrac{7}{2}\right\}$

   e. $\left\{x : x \leq -\dfrac{1}{2} \text{ or } x \geq \dfrac{7}{2}\right\}$

12. What is the value of $(g \circ f)(1)$ if $f(x) = \dfrac{3}{x-3}$ and $g(x) = \dfrac{3}{x}$?

   a. $\dfrac{1}{2}$

   b. 2

   c. undefined

   d. $-\dfrac{1}{2}$

   e. -2

13. What is the vertical asymptote of $f^{-1}(x)$ if $f(x) = \dfrac{x+3}{x-2}$?

   a. $y = 1$

   b. $x = 2$

   c. $y = 2$

   d. $x = 1$

   e. $x = -1$

14. Which of the following is a recursive formula for the sequence 38, 49, 60, 71,...?
   a. $a_{n+1} = a_n + 11, a_1 = 38$
   b. $a_n = a_n + 11, a_1 = 38$
   c. $a_{n+1} = a_n - 11, a_1 = 38$
   d. $a_{n+1} = 11a_n, a_1 = 38$
   e. $a_n = a_n - 11, a_n = 38$

15. Which of the following is an equation of the line perpendicular to $2x - 3y = 8$ and passes through the point $(-6, 3)$?
   a. $3x - 2y = -12$
   b. $3x + 2y = -12$
   c. $2x + 3y = 12$
   d. $2x - 3y = -12$
   e. $3x + 2y = 12$

16. What are the center and radius of the circle with equation $x^2 + y^2 + 4x + 6y = 3$?
   a. Center: $(-2, -3)$, radius: $r = 4$
   b. Center: $(2, -3)$, radius: $r = 4$
   c. Center: $(-2, 3)$, radius: $r = 16$
   d. Center: $(2, 3)$, radius: $r = 4$
   e. Center: $(-2, -3)$, radius: $r = 16$

17. What are all the zeros of $f(x) = 3x^3 + 4x^2 - 17x - 6$ given that $x = 2$ is a zero?
   a. $x = 2, 3, \dfrac{1}{3}$
   b. $x = 2, -3, \dfrac{1}{3}$
   c. $x = 2, 6, \dfrac{1}{3}$
   d. $x = 2, 3, -\dfrac{1}{3}$
   e. $x = 2, -3, -\dfrac{1}{3}$

18. Consider the function and graph below. Is the function even, odd or neither even nor odd and what symmetry does the function does have?

$$f(x) = -3x^3 + 6x$$
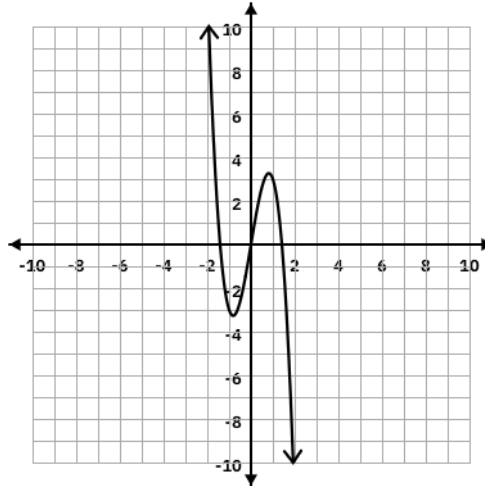
a. The function is even and is symmetric with respect to the origin.
b. The function is even and is symmetric with respect to the y-axis.
c. The function is odd and is symmetric with respect to the origin.
d. The function is odd and is symmetric with respect to the x-axis.
e. The function is neither even nor odd but is symmetric with respect to the origin.

19. What are the natures of the solutions of the quadratic equation $-6x^2 + 3 = -4x$?
   a. There will be one real number solution.
   b. There will be two solutions with imaginary numbers.
   c. There will be one real number solution and one imaginary number solution.
   d. There will be two different real number solutions.
   e. There is no solution to this equation.

20. What is the equation of the parabola with a focus of $(5, 2)$ and a directrix of $x = -1$?
   a. $x = \frac{1}{12}(y - 2)^2 + 2$
   b. $y = \frac{1}{2}(x - 2)^2 + 2$
   c. $x = -\frac{1}{12}(y - 2)^2 + 2$
   d. $x = \frac{1}{4}(y - 2)^2 + 2$
   e. $x = (y - 2)^2 + 2$

21. What is the 20th term and the sum of the first twenty terms of the sequence that begins
3, 6, 12, 24, ... ?
   a. The 20th term is 3,145,725 and the sum of the first twenty terms is 1,572,864.
   b. The 20th term is 1,572,864 and the sum of the first twenty terms is 3,145,725.
   c. The 20th term is 3,145,725 and the sum of the first twenty terms is 3,254,554.
   d. The 20th term is 247,726 and the sum of the first twenty terms is 1,572,864.
   e. The 20th term is 1,572,864 and the sum of the first twenty terms is 5,204,118.

22. What are all values of x that satisfy $y_1 - y_2 = y_3$ if $y_1 = 3x + 6$, $y_2 = 4(x - 2) + 3$ and $y_3 = 7$?
   a. 4
   b. -4
   c. 12
   d. -12
   e. 1

23. What is the solution set to the inequality $\frac{(x+7)(x-8)}{x-3} \geq 0$?
   a. $\{x: -7 \leq x < 3\}$
   b. $\{x: -7 \leq x \leq 3 \text{ or } x \geq 8\}$
   c. $\{x: -7 \leq x < 3 \text{ or } x \geq 8\}$
   d. $\{x: x \geq 8\}$
   e. $\{x: x < 3 \text{ or } x \geq 8\}$

24. What is the product, BA, if it is defined, if $A = \begin{bmatrix} -1 & 0 & 1 \\ -4 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$?
   a. $\begin{bmatrix} -4 & -1 & -3 \\ -10 & 3 & 7 \end{bmatrix}$
   b. $\begin{bmatrix} 4 & -1 & 3 \\ -10 & 3 & 7 \end{bmatrix}$
   c. The product is not defined.
   d. $\begin{bmatrix} 0 & -1 & -3 \\ -10 & 0 & 7 \end{bmatrix}$
   e. $\begin{bmatrix} -4 & 1 & 3 \\ 10 & -3 & -7 \end{bmatrix}$

25. What value(s) for x that satisfy the equation $\log_5(x + 6) + \log_5(x - 6) - \log_5 x = 1$?
   a. -9
   b. 9, -4
   c. 4
   d. 9
   e. 9, 4

26. Suppose a security passcode is comprised of five digits from 0-9. The first digit cannot be a zero and the third digit must be a four. If numbers can be repeated in the code, how many possible five-digit codes can there be?
   a. 10000
   b. 9000
   c. 5800
   d. 10250
   e. 9580

27. Let the revenue and cost functions for a company be given by $R(x) = 400x - 0.25x^2$ and $C(x) = 10x + 7$, respectively, where x is the number of units produced and $R(x)$ and $C(x)$ are in thousands of dollars. What is the number of units that must be produced and sold to obtain the maximum profit?
   a. 700
   b. 780
   c. 1560
   d. 1025
   e. 854

28. Suppose Ali has 53 coins in a bag comprised of only quarters and dimes totaling $10.85. How many dimes does Ali have in the bag?
   a. 37 dimes
   b. 41 dimes
   c. 12 dimes
   d. 16 dimes
   e. 34 dimes

29. What is the exact value for x that satisfies the equation $3^{x-2} = 7^{x+6}$?
   a. $\dfrac{2\ln (3)+6\ln (7)}{\ln(3)-\ln (7)}$
   b. $\dfrac{2\ln(3)-6\ln (7)}{\ln(3)+\ln (7)}$
   c. $\dfrac{\ln(3)-6\ln (7)}{\ln(3)-\ln (7)}$
   d. $\dfrac{2\ln (3)+6\ln (7)}{4\ln(3)-3\ln (7)}$
   e. $\dfrac{\ln (3)+\ln (7)}{2\ln(3)-6\ln (7)}$

30. What is $\dfrac{x^2+3x-18}{x^2-5x+6} \div \dfrac{x^2+7x+6}{2+x-x^2}$ simplified completely?

   a.   1

   b. $\dfrac{x+6}{x-7}$

   c. -1

   d. $-\dfrac{1}{x+1}$

   e. $x-4$

31. What is the simplified form of the following expression using only positive exponents?

$$\left(\dfrac{4x^3y^2z^9}{3x^5z^{11}}\right)^{-3}$$

   a. $\dfrac{64x^6z^6}{27y^6}$

   b. $\dfrac{27x^{24}z^{18}}{64y^6}$

   c. $\dfrac{64x^6z^6}{27}$

   d. $\dfrac{3x^6z^6}{4y^6}$

   e. $\dfrac{27x^6z^6}{64y^6}$

32. Six less than a number is at most the same number divided by 2. What is the largest possible value of the number?

   a.   10

   b.   4

   c.   6

   d.   12

   e.   14

33. Which value(s) for x satisfy the equation $4x^{-2} + 5x^{-1} + 1 = 0$?
    a. $x = 1, 4$
    b. $x = -1, -4$
    c. $x = -1$
    d. $x = -4$
    e. $x = 1, -4$

34. How many liters of a 20% chlorine solution should be mixed with 30 liters of a 50% chlorine solution to get a mixture that is 30% chlorine solution?
    a. 60 liters
    b. 30 liters
    c. 45 liters
    d. 50 liters
    e. 70 liters

35. What is the other root of the equation $x^2 + x + c = 0$ if $x = 3$ is a root?
    a. $x = 4$
    b. $x = -3$
    c. $x = -4$
    d. $x = 2$
    e. $x = -2$

36. Let $f(x)$ be a one-to-one function with a domain of $[-4, 3)$ and a range of $[0, \infty)$. What is the range of $f^{-1}(x)$?
    a. $[-4, 3)$
    b. $[0, \infty)$
    c. $(-\infty, \infty)$
    d. $(-4, 3]$
    e. $(0, \infty)$

37. A light pole that is 15 feet tall casts a shadow that is 18 feet long. What is the length of the shadow from a light pole that is 10 feet tall?
    a. 14 feet
    b. 10 feet
    c. 12 feet
    d. 6 feet
    e. 18 feet

38. Which of the following is equivalent to the trigonometric expression $\dfrac{\sec(x)\sin(x)}{\tan(x)+\cot(x)}$?

   a. $\sin^2(x)$
   b. $\cos^2(x)$
   c. $\sin(x)$
   d. $\cos(x)$
   e. 1

39. Suppose P is directly proportional to q and inversely proportional to r^2. When $q = 1$ and $r = 2$, $P = 4$. What is the value of P if $q = 2$ and $r = 4$?

   a. 4
   b. 8
   c. 16
   d. 2
   e. 6

40. What is the value of $a + b$ if $(2,1,4)$ is a solution of the following system of equations?

$$
\begin{aligned}
x + y + z &= 7 \\
3x - 2y + bz &= -4 \\
-2x + ay - 5z &= -20
\end{aligned}
$$

   a. 2
   b. -2
   c. 6
   d. -6
   e. 0